INFORMATION ON MASTER'S THESIS

1. Full name: Ngô Anh Tuấn

2. Sex: male

3. Date of birth: 02-11-1988

4. Place of birth: Hoa Binh

5. Admission decision number: Dated

6. Changes in academic process: change major from *Geometry and Topology* to *Algebra and Number theory*

7. Official thesis title:

Some problems on the algebraic version of the classical conjecture on spherical classes

8. Major: Algebra and Number theory

9. Code: 604605

10. Supervisors: Prof. Nguyễn Hữu Việt Hưng, Department of Mathematics, Ha Noi University of Science

11. Summary of the finding of the thesis:

The thesis is divided into three chapters.

-In chapter 1, we recall the definition and some properties of the Steenrod algebra, $\mathcal{A}$, over the field of 2 elements, $\mathbb{F}_2$. We exploit Singer's invariant theoretic description of the lambda algebra. We have

$$D_k = \mathbb{F}_2[Q_{k,0}, \ldots, Q_{k,k-1}],$$

where $Q_{k,i}$ denotes the Dickson invariant of degree $2^k - 2^i$. As the action of GL_k and $\mathcal{A}$ on P_k commute with each other, the Dickson algebra, D_k, is an algebra over $\mathcal{A}$. Singer sets $\Gamma_k = D_k[Q_{k,0}^{-1}]$, the localization of D_k given by inverting $Q_{k,0}$, and defines $\Gamma_k^\wedge$ to be a certain "not too large" submodule of Γ_k. He also equips $\Gamma^\wedge = \oplus_k \Gamma_k^\wedge$ with a differential $\partial : \Gamma_k^\wedge \longrightarrow \Gamma_{k-1}^\wedge$ and a coproduct. Then, he shows that the differential coalgebra $\Gamma^\wedge$ is dual to the (opposite) lambda algebra. Thus, $H_k(\Gamma^\wedge) \cong Tor_k^{\mathcal{A}}(\mathbb{F}_2, \mathbb{F}_2)$.

-In chapter 2, we recall the definition of the Lannes-Zarati homomorphism and the algebraic version of the classical conjecture on spherical classes. We consider the Hurewicz homomorphism $H : \pi_*^s(S^0) \cong \pi_*(Q_0S^0) \longrightarrow H_*(Q_0S^0)$, where Q_0S^0 be the basepoint component of $QS^0 = \lim_n \Omega^n S^n$. The classical conjecture on spherical classes states that the Hopf invariant one and the Kervaire invariant one classes are the only elements in $H_*(Q_0S^0)$ detected by the Hurewicz homomorphism. In 1987, Lannes and Zarati construct homomorphisms

$$\varphi_k : Ext_{\mathcal{A}}^{k,k+i}(\mathbb{F}_2, \mathbb{F}_2) \longrightarrow (\mathbb{F}_2 \underset{\mathcal{A}}{\otimes} D_k)_i^*,$$

which correspond to an associated graded of the Hurewicz map. The Hopf invariant one and the Kervaire invariant one classes are respectively represented by certain permanent cycles in $Ext_{\mathcal{A}}^{1,*}(\mathbb{F}_2, \mathbb{F}_2)$ and $Ext_{\mathcal{A}}^{2,*}(\mathbb{F}_2, \mathbb{F}_2)$, on which φ_1 and φ_2 are non-zero. Nguyễn H. V. Hưng present the algebraic version of the classical conjecture on spherical classes which states that $\varphi_k = 0$ in any positive stem i for $k > 2$. This conjecture has been proved for $k = 3$ and 4 by Nguyễn H. V. Hưng.

We also recall a proposition of Hưng which states that the Lannes-Zarati map

$$\varphi = \bigoplus_k \varphi_k : \bigoplus_k Ext_{\mathcal{A}}^k(\Sigma^{-k}\mathbb{F}_2, \mathbb{F}_2) \longrightarrow \bigoplus_k (\mathbb{F}_2 \underset{\mathcal{A}}{\otimes} D_k)^*$$

is an algebra homomorphism.

That φ_k vanishes for $k > 2$ on the decomposable elements in $Ext_{\mathcal{A}}^k(\mathbb{F}_2, \mathbb{F}_2)$ was proved by F. P. Peterson and Nguyễn H. V. Hung. Besides, φ_k vanishes on the Singer transfer's image was proved by Nguyễn H. V. Hung and Trần N. Nam.

We present the proof of Hưng's theorem which states that the inclusion $D_k \subset \Gamma_k^\wedge$ is a chain level representation of the Lannes-Zarati homomorphism. An immediate consequence of this theorem is the equivalence between the algebraic version of the classical conjecture on spherical classes and the following conjecture which states that if $q \in D_k^+$, then $[q] = 0$ in $Tor_k^{\mathcal{A}}(\mathbb{F}_2, \mathbb{F}_2)$ for $k > 2$.

-In chapter 3, we recall the definition of the squaring operations. Liulevicius construct squaring operations

$$Sq^i : Ext_{\mathcal{A}}^{s,s+d}(\mathbb{F}_2, \mathbb{F}_2) \longrightarrow Ext_{\mathcal{A}}^{s+i,2(s+d)}(\mathbb{F}_2, \mathbb{F}_2),$$

which share most of the properties with Sq^i on the cohomology of spaces. However, Sq^0 is not the identity. Nguyễn H. V. Hưng have defined a squaring operation

$$Sq^0 : P(\mathbb{F}_2 \underset{GL_k}{\otimes} H_*(B\mathbb{V}_k))_d \longrightarrow P(\mathbb{F}_2 \underset{GL_k}{\otimes} H_*(B\mathbb{V}_k))_{2d+k},$$

which is derived from Kameko's squaring operation Sq^0 on $\mathbb{F}_2 \underset{GL_k}{\otimes} PH_*(B\mathbb{V}_k)$.

We recall Hưng's theorem which states that the squaring operation Sq^0 on $(\mathbb{F}_2 \underset{\mathcal{A}}{\otimes} D_k)^*$ commutes with the classical squaring operation Sq^0 on $Ext_{\mathcal{A}}^k(\mathbb{F}_2, \mathbb{F}_2)$. In other words, the following diagram commutes

$$\begin{array}{ccc} Ext_{\mathcal{A}}^k(\mathbb{F}_2, \mathbb{F}_2) & \xrightarrow{\varphi_k} & P(\mathbb{F}_2 \underset{GL_k}{\otimes} H_*(B\mathbb{V}_k)) \\ \downarrow{\scriptstyle Sq^0} & & \downarrow{\scriptstyle Sq^0} \\ Ext_{\mathcal{A}}^k(\mathbb{F}_2, \mathbb{F}_2) & \xrightarrow{\varphi_k} & P(\mathbb{F}_2 \underset{GL_k}{\otimes} H_*(B\mathbb{V}_k)). \end{array}$$

Using the above theorem and results of Lin, Chen, Peterson and Giambalvo, we give a new proof for the algebraic version of the classical conjecture on spherical classes for $k = 3$ and 4. Moreover, we prove this conjecture for $k = 5$.

In this thesis, we give a new proof for the algebraic version of the classical conjecture on spherical classes for $k = 3$ and 4. Futhermore, we prove that the Lannes-Zarati homomorphism of rank 5 vanishes in any positive stems.

The main result of this thesis is that the Lannes-Zarati homomorphism of rank 5 vanishes in any positive stems. This establishes a part of the algebraic version of the classical conjecture on spherical classes.

Date:

Signature:

Fullname: