INFORMATION ON DOCTORAL THESIS

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11. Summary of the new findings of the thesis

a. Thesis purpose and objectives

To describe complex systems in nature, economics, energy, aviation, etc., single, individual systems are often inadequate, and instead, multiple subsystems must be combined with specific constraints. A switched system consists of a finite number of subsystems and the switching rules among them. The subsystems can be continuous or discrete, singular or non-singular. The switching rule is a piecewise constant function that depends on time variables, past values, the state $x(t)$ of each individual subsystem, or random switching with a given distribution function.

In fact, switching may occur due to sudden, unpredictable changes within the system, such as the failure of a system component or the accidental activation of a subsystem. In such cases, to ensure the system's safety, it must be designed to remain stable under all switching rules. In a switched system, the switching signal is often hidden and uncontrollable. Thus, the stability of the switched system essentially refers to its robustness against all switching disturbances. One of the main problems in studying switched systems is to determine the conditions under which the system remains stable regardless of the switching rule. Additionally, some subsystems may be unstable, requiring the design of specific switching sequences to stabilize the overall system—this is known as the stabilization problem for switched systems.

Consider a switched discrete-time linear singular (SDLS) system of the form

$$E_{\sigma(k)}x(k+1) = A_{\sigma(k)}x(k), \tag{1}$$

where $E_i, A_i \in \mathbb{R}^{n\times n}$, $x(k) \in \mathbb{R}^n$ and $\sigma : \mathbb{N} \cup \{0\} \to \underline{N} := \{1, 2, \ldots, N\}, N \in \mathbb{N}$, denotes the switching signal that determines which of the $j \in \underline{N}$ modes is active at time k. Suppose that the matrices E_i are singular for all $i \in \underline{N}$.

To our knowledge, there are still quite few results on positive SDLS (1). Therefore, we aim to investigate the positivity, stability, and stabilizability of positive switched discrete–time linear singular systems of the form (1). We also impose an index-1 condition on system (1), a requirement related to causality in response to switching signals, meaning that changes in the switching signal in the future do not alter the solution at the current (or past) time. Using a one-step mapping and Lyapunov-like stability conditions, we study the positivity and stability of the index-1 SDLS. Then, by extending Fekete's lemma, we define the joint spectral subradius for a family of matrix pairs $\{(E_i, A_i)\}_{i=1}^{N}$, providing characteristics for the stabilizability of the positive SDLS.

On the other hand, recent research by P.K. Anh and colleagues has investigated the solvability and stability of SDLS systems of the form (1), where the switching rules in matrices E and A are identical. In fact, systems may be subject to unwanted disturbances. Therefore, we aim to study the solvability and stability of switched discrete–time linear singular systems with Lipschitz perturbations. Furthermore, if the switching rules in matrices E and A are different, the problem becomes more complicated. This occurs when the dynamics of x_{k+1} depend on the leading matrix E at time $k+1$, such as when discretizing the system using the implicit Euler method. P.T. Linh has proposed some results for this case with undisturbed SDLS. To our knowledge, there are no existing results on the solvability of SDLS under Lipschitz perturbations $f_{\sigma(k)}(x(k))$. Thus, we study the solvability and stability of switched discrete–time linear singular systems under Lipschitz perturbations in two cases: Case 1, where the switching rules in matrices E and A are identical in form

$$E_{\sigma(k)}x(k+1) = A_{\sigma(k)}x(k) + f_{\sigma(k)}(x(k)) \tag{2}$$

and Case 2 for the switching rules in matrices E and A are different, i.e.,

$$E_{\sigma(k+1)}x(k+1) = A_{\sigma(k)}x(k) + f_{\sigma(k)}(x(k)) \tag{3}$$

where $\sigma : \mathbb{N} \cup \{0\} \to \underline{N}$, is a switching signal taking values in the finite set $\underline{N}$, $E_i, A_i \in \mathbb{R}^{n\times n}$ and $f_i : \mathbb{R}^n \to \mathbb{R}^n, i \in \underline{N}$, are perturbations, $x(k) \in \mathbb{R}^n$ is state vector at time $k \in \mathbb{N}$. Suppose that the matrices E_i are singular for all $i \in \underline{N}$. The different switching rules in coefficient matrices E and A in (3) combined with the dynamics of (3) and the integration of singular subsystems, cause some difficulties in studying the system's solvability and stability. The unique existence of a solution to (3) will be demonstrated by using the contraction mapping principle. After that, stability characteristics of (3) will be established by using the Lyapunov function method, solution estimates, and the discrete Gronwall inequality.

b. Research methods

To study the solvability, stability, and stabilizability of switched discrete–time linear singular systems, we utilize not only some fundamental properties of calculus, functional analysis, and linear algebra, such as the contraction mapping principle, comparison principle, matrix norm properties, and properties of sequences and function series, but also methods like projection, the Lyapunov function approach, spectral radius, and spectral subradius for a family of matrix pairs. All numerical calculations were performed in the Matlab environment on a personal computer with a Core i5 processor and 8GB RAM.

c. Major results of thesis

In this thesis, we have studied and solved two problems related to switched discrete–time linear singular systems.

Problem 1 investigates positive switched discrete–time linear singular systems of index-1. We studied the stability of these systems through the linear programming (LP) method along with the properties of index-1 systems, establishing stability conditions with minimal dwell time. Then, we introduce a notion of joint spectral subradius of a finite set of matrix pairs, which allows us to fully characterize stabilizability.

Problem 2 focuses on the solvability and stability of a class of switched discrete–time linear singular systems with Lipschitz perturbations, where the homogeneous systems have index-1, under two cases: switching rules are identical or different for the coefficient matrix pairs. We present results on the solvability of this class of systems by using projection properties, the properties of index-1 homogeneous systems, and the contraction mapping principle. Then, the thesis establishes conditions for stability, uniform stability, and asymptotic stability of these systems based on the Lyapunov function method. Finally, we propose exponential stability conditions by applying the variation of constants formula for solutions, estimating solutions, and using the discrete Gronwall inequality.

12. Futher research directions

Some directions for a future research

- Study the solvability and stability of nonlinear singular discrete switched systems of the form $E_{\sigma(k)}x(k+1) = F_{\sigma(k)}(x(k))$.
- Stabilize switched discrete-time linear singular systems of index-1 using appropriate switching rules or feedback controls.
- Investigate the stability and stabilizability of singular discrete switched systems without assuming an index-1 condition.

13. Publications related to the dissertation

[CT1] D.D. Thuan, **N.T. Thu** (2024), *Stability and stabilizability of positive switched discrete-time linear singular systems*, Systems & Control Letters, Vol.185, article number 105725, https://doi.org/10.1016/j.sysconle.2024.105725.

[CT2] D.D. Thuan, **N.T. Thu** (2024), *Solvability and stability of switched discrete time linear singular systems under Lipschitz perturbations*, Journal of Difference Equations and Applications, 2024, Vol. 30, No. 7, pp. 849–869, https://doi.org/10.1080/10236198.2024.2336478.

[CT3] **N.T.Thu** (2024), *Solvability and Stability of Switched Discrete–time Singular Systems with the Same Switching Rules in Coefficient Matrices*, VNU Journal of Science: Mathematics – Physics, Vol. 40, No. 2, 106–115.

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On behalf of academic supervisors **PhD. Student**