

INFORMATION ON DOCTORAL THESIS

1. Full name: Nguyen Hai Ha
2. Sex: male
3. Date of birth: April 24, 1989
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5. Admission decision number 3613/QD-DHKHTN dated November 30, 2020 by VNU University of Science
6. Changes in academic process:
 - Decision No. 3156/QD-DHKHTN dated November 17, 2021 on adjusting the supervisory team for PhD student of the course QH.2020.T.NCS;
 - Decision No. 4955/QD-DHKHTN dated December 26, 2023 on adjusting the time of doctoral programs;
 - Decision No. 4828/QD-DHKHTN dated December 31, 2024 on extending training and thesis defense for doctoral students.
7. Official thesis title: Some regularization methods for solving monotone inclusion problems and equilibrium problems
8. Major: Applied mathematics
9. Code: 94060112.01
10. Supervisors:

Main supervisor: Dr. Dang Van Hieu
Workplace: Air Force Officer's College, Nha Trang, Khanh Hoa

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11. Summary of the new findings of the thesis

11.1. Thesis purpose and objectives

The objective of the thesis is to study and propose some regularization methods to solve monotone variational inclusion problems in real Hilbert spaces and equilibrium problems, specifically as follows:

- Construct a regularization proximal and contraction method to solve variational inclusion (VI) problems;
- Combine the above method with multi-step inertial technique to achieve the new method with accelerated convergence rate;

- Construct a regularization proximal-like method to solve split variational inclusion (SVI) problems;
- Construct a regularized dynamical system of extragradient type to solve equilibrium problems (EP) and then construct a new iterative method to solve EP by discretizing the received dynamical system.

11.2. Research methods

This thesis improves and integrates known methods to achieve new algorithms, not too complicated but efficient. The tools that the thesis is interested in are the contraction and proximal method, the regularization technique, the inertial technique, the extragradient method, and the dynamical system method. In addition, the thesis is also interested in the strategy of choosing the appropriate stepsizes for the algorithm based on the criteria called the predicted step size conditions (PSC). The proposed methods will be applied to some specific problems. Finally, the thesis performs some numerical experiments to illustrate the effectiveness of the obtained methods and compare them with other known methods.

11.3. Main results, new contributions of the thesis and conclusions

The main results obtained from the thesis include:

- Regularization proximal and contraction method (RPCM) for solving VI problems and its convergence;
- Regularization proximal-like algorithm (RPA) for solving SVI problems and its convergence;
- First-order regularized dynamical system of extragradient type for solving EPs and the strong convergence of its trajectory. Discretizing this dynamical system, we obtain a relaxed regularized extragradient method for solving EPs and its convergence.

In terms of significance and importance, the RPCM and RMSIPM methods provide new tools for solving the VI problem. Compared with many other methods for solving this problem, the two methods we propose have the advantage of strong convergence in infinite-dimensional Hilbert spaces and do not considerably change the structure of the original method (which is, the proximal contraction method). The RPA method for solving the SVI problem also has some advantages when compared with some other nearby methods, such as the method of Chuang (2016). In addition to strong convergence, the RPA method only requires the solution of $J_{\beta}^{B_1}$ once at each iteration, which helps to reduce the computational cost. Next, the regularized dynamical system of extragradient type provides a continuous method with less strict conditions on the parameters, which has a trajectory converging strongly to the correct solution of the EP problem. The REM method constructed from the discretization of the above dynamical system is also a strongly convergent method in the infinite-dimensional Hilbert space. When compared with some other strongly

convergent methods, the RPCM, RMSIPM, RPA, and REM methods also converge faster in the specific numerical experiments that the thesis builds.

12. Further research directions:

- Extend the methods for solving monotone variational problems to the more general case of pseudomonotone variational inclusion problems with an additive structure;
- Extend the iterative methods studied in the thesis to the case where the data are random;
- Study the convergence rates of iterative methods presented in the thesis;
- Propose self-adaptive strategy for choosing the bounded stepsize function used in the dynamical system in the case of unknown Lipschitz-type constants of the cost bifunction.

13. Thesis-related publications

- Hieu D.V., Anh P.K., Ha N.H. (2021), “Regularization proximal method for monotone variational inclusions”, *Networks and Spatial Economics* 21, pp. 905-932 (SCIE).
- Hieu D.V., Reich S., Anh P.K., Ha N.H. (2022), “A new proximal-like algorithm for solving split variational inclusion problems”, *Numerical Algorithms* 89, pp. 811-837 (SCIE).
- Hieu D.V., Cho Y.J., Quy P.K., Ha N.H. (2022), “An effective iterative projection method for variational inequalities in Hilbert spaces”, *Soft Computing* 26, pp. 10207-10221 (SCIE).
- Ha N.H., Hieu D.V., Muu L.D. (2024), “An accelerated regularization method for variational inclusions”, *Journal of Applied and Numerical Optimization* 6, pp. 97-113 (SCOPUS).
- Hieu D.V., Ha N.H. (2024), “The relaxed regularized method of extragradient type for equilibrium problems”, *Applied Set-Valued Analysis and Optimization* 6, pp. 385-412 (SCOPUS).

Date:

On behalf of academic supervisors

PhD. Student